HALE
SCHOOL

Semester Two Examination, 2019

Question/Answer booklet

Year 11

MATHEMATICS METHODS
UNITS 1 AND 2

Section One:
Calculator-free

SOLUTIONS

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	90	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free **35% (50 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(4 marks)**

The line segment between the points $A(3,2)$ and $B(3,-4)$ is the diameter of a circle.

Determine the equation of the circle in the form $x^2+ax+y^2+by=c$, where a, b and c are constants.

Solution
Centre: $\left(3, \frac{2-4}{2}\right) = (3,-1)$ Radius: $r = 2 - -1 = 3$ Equation: $(x-3)^2 + (y+1)^2 = 3^2$ $x^2 - 6x + 9 + y^2 + 2y + 1 = 9$ $x^2 - 6x + y^2 + 2y = -1$
Specific behaviours
✓ centre calculated correctly ✓ radius stated ✓ factored equation with centre and radius substituted ✓ correct equation

Question 2 **(5 marks)**

Determine the gradient of the curve $y = x^2 - 3x - 40$ at the point(s) where it crosses the x-axis.

Solution
$(x+5)(x-8)=0$ $x=-5, x=8$ $\frac{dy}{dx}=2x-3$ $x=-5, \frac{dy}{dx}=-13$ $x=8, \frac{dy}{dx}=13$ At $(-5,0)$ gradient is -13 and at $(8,0)$ gradient is 13.
Specific behaviours
✓ factorises quadratic correctly or substitutes into quadratic formula ✓ determines roots ✓ derives quadratic correctly ✓ calculates one point and gradient at that point ✓ calculates second point and gradient at that point (no need for coordinates) but must clearly match x-value to the its gradient.

Question 3 **(8 marks)**

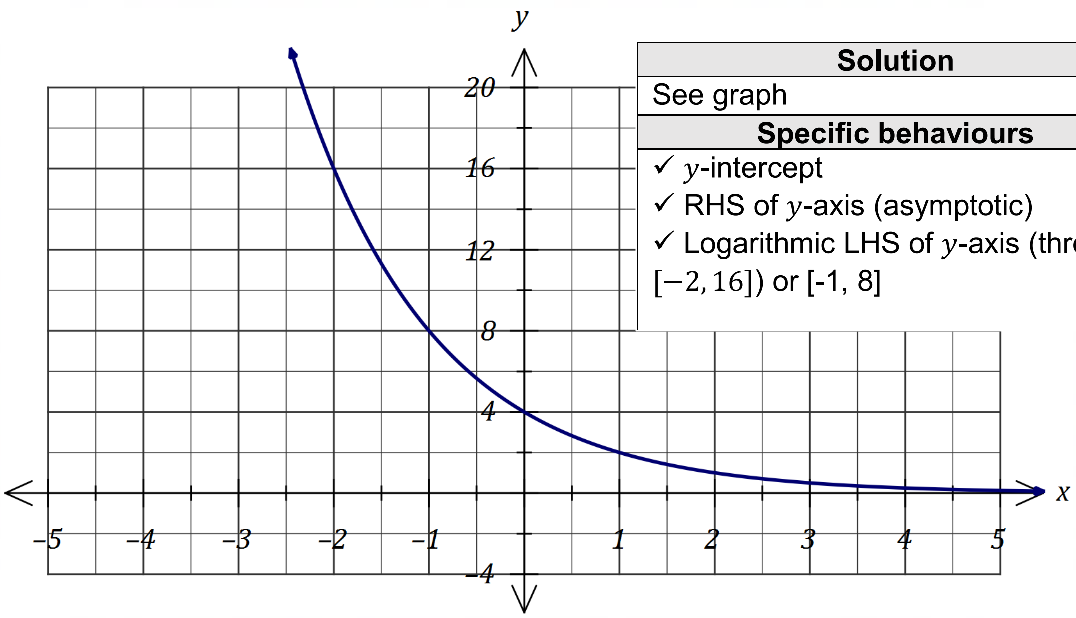
(a) Simplify $(2t - 5\sqrt{t})(2t + 5\sqrt{t})$. (2 marks)

Solution
$(2t - 5\sqrt{t})(2t + 5\sqrt{t}) = (2t)^2 - \left(5\sqrt{t}\right)^2$ $= 4t^2 - 25t$
Specific behaviours
✓ indicates use of difference of squares or expands correctly ✓ correct simplification

(b) Solve the equation $9^{2x} = \frac{\sqrt{3}}{81}$ for x. (3 marks)

Solution
$(3^2)^{2x} = 3^{0.5} \times 3^{-4}$ $3^{4x} = 3^{-3.5}$ $4x = -3.5$ $x = -0.875 = -\frac{7}{8}$
Specific behaviours
✓ writes 9 and 81 as powers of 3 ✓ simplifies RHS using correct index laws ✓ correct solution (decimal in fraction not accepted)

(c) Sketch the graph of $y = 4 \times 2^{-x}$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ y-intercept ✓ RHS of y-axis (asymptotic) ✓ Logarithmic LHS of y-axis (through $[-2, 16]$) or [-1, 8]

Question 4 **(7 marks)**

Small body A is moving along a straight line so that at any time t seconds, its displacement relative to a fixed point O on the line is given by $x = t^3 - 6t^2 + 2$ cm.

(a) Determine the velocity of A when $t = 2$. (2 marks)

Solution
$v = \frac{dx}{dt} = 3t^2 - 12t$ $v(2) = 3(2)^2 - 12(2)$ $= -12 \text{ cm/s}$
Specific behaviours
✓ expression for velocity ✓ correct velocity with units

(b) Determine the displacement of A relative to O at the instant(s) that it is stationary. (3 marks)

Solution
$3t^2 - 12t = 0$ $3t(t-4) = 0$ $t = 0, t = 4$ $x(0) = 2 \text{ cm}, \quad x(4) = -30 \text{ cm}$
Specific behaviours
✓ factorises velocity and/or finds both values of t ✓ one correct displacement ✓ both correct displacements

Small body B has velocity given by $v = 3t^2 - 8t + 2$ cm/s and when $t = 1$ it has a displacement of 7 cm relative to O.

(c) Determine an expression for the displacement of B relative to O at any time t. (2 marks)

Solution
$\frac{dx}{dt} = 3t^2 - 8t + 2$ $x = t^3 - 4t^2 + 2t + c$ $c = 7 - (1 - 4 + 2) = 8$ $x = t^3 - 4t^2 + 2t + 8$
Specific behaviours
✓ antidifferentiates ✓ correct expression

Question 5 **(7 marks)**

(a) Using Pascal's triangle, or otherwise, determine $\binom{4}{2}$. (1 mark)

Solution
$\binom{4}{2} = 6$
Specific behaviours
✓ correct value

(b) Expand $(x+1)^4$. (2 marks)

Solution
$(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$
Specific behaviours
✓ correct coefficients ✓ correct expansion

(c) Hence, or otherwise, determine the equation of the tangent to the curve $y = (x+1)^4$ at the point where $x = -2$. (4 marks)

Solution
$\frac{dy}{dx} = 4x^3 + 12x^2 + 12x + 4$ When $x = -2$ $y = (-1)^4 = 1$ $\frac{dy}{dx} = -32 + 48 - 24 + 4 = -4$ Hence equation of tangent is $y - 1 = -4(x+2)$ or $y = -4x - 7$
Specific behaviours
✓ derivative correct ✓ finds y-coordinate ✓ calculates gradient ✓ equation of tangent (any form)

Question 6 **(8 marks)**

(a) Solve the following equations.

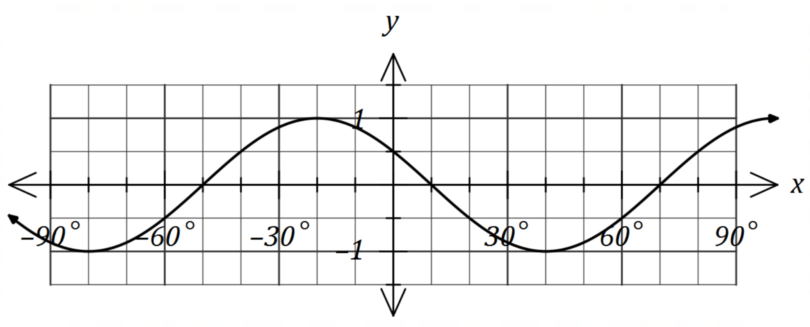
(i) $\tan(3x) = \sqrt{3}$, $0 \le x \le \pi$. (2 marks)

Solution
$3x = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}$ $x = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$
Specific behaviours
✓ one correct solution ✓ all correct solutions

(ii) $2\sin(x - 60°) = \sqrt{3} + \sin x$, $0° \le x \le 360°$. (4 marks)
(hint: expand the left-hand side of the equation)

Solution
$2(\sin x \cos 60° - \cos x \sin 60°) = \sqrt{3} + \sin x$ $2\left(\sin x\left(\frac{1}{2}\right) - \cos x\left(\frac{\sqrt{3}}{2}\right)\right) = \sqrt{3} + \sin x$ $\cos x = -1$ $x = 180°$
Specific behaviours
✓ uses angle difference identity ✓ substitutes exact values correctly ✓ simplifies equation ✓ correct solution

(b) The graph of $y = \cos(ax + b)$ is shown below, where a and b are positive constants.



Determine the minimum possible value of each of the constants. (2 marks)

Solution
Period of $120° \Rightarrow a = 360° \div 120° = 3$ $y = \cos(3(x + 20)) = \cos(3x + 60°)$ $a = 3, \quad b = 60°$
Specific behaviours
✓ value of a ✓ value of b

Question 7 **(4 marks)**

Show that $x = 1$ is the **only** stationery point on the curve $y = x^4 + 4x^2 - 12x + 20$.

Solution
$\frac{dy}{dx} = 4x^3 + 8x - 12$ $4x^3 + 8x - 12 = 0$ $x^3 + 2x - 3 = 0$ $(x-1)(x^2 + ax + 3) = 0$ $(x-1)(x^2 + x + 3) = 0$ $b^2 - 4ac = 1 - 4(1)(3) = -11 \Rightarrow \text{No solutions}$
Specific behaviours
✓ derivative correct and equated to 0 ✓ shows that $(x-1)$ is a factor or substitutes to show x=1 is a stationary point ✓ finds the remaining factor $x^2 + x + 3$ ✓ uses discriminant to show quadratic has no solutions

Question 8 **(7 marks)**

An arithmetic sequence has an explicit rule $T_n = 52 - 4n$.

(a) Use a recursive rule to express the sequence. (2 marks)

Solution
$T_{n+1} = T_n - 4$ $T_1 = 48$ or $T_0 = 52$
Specific behaviours
✓ $T_{n+1} = T_n - 4$ ✓ $T_1 = 48$

(b) Determine S_{10}. (2 marks)

Solution
$S_{10} = \frac{10}{2}\left((2)(48) + (9)(-4)\right)$ $S_{10} = 300$
Specific behaviours
✓ correct substitution ✓ state the sum

(c) The sum of the first m terms of the sequence is 200. Determine the value(s) of the integer constant m. (3 marks)

Solution
$200 = \frac{m}{2}\left(2(48) + (m-1)(-4)\right)$ $400 = m(96 - 4m + 4)$ $4m^2 - 100m + 400 = 0$ $m^2 - 25m + 100 = 0$ $(m-5)(m-20) = 0$ $m = 5, \quad m = 20$
Specific behaviours
✓ substitutes into sum formula ✓ simplifies and equates quadratic to zero ✓ both correct solutions

End of questions

Supplementary page

Question number: _________